8.3 Relative Rates of Growth

Definitions: Faster, Slower, Same-rate Growth

$f(x)$ and $g(x)$ grow at the same rate if $\lim_{x\to\infty} \frac{f(x)}{g(x)} = c$ $\quad c \neq 0$

$\lim_{x\to\infty} \frac{f(x)=10x}{g(x)=2x} = 5$

$10x$ & $2x$ grow at same rate (same order of magnitude)

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compare

$\underset{f(x)}{10x}, \underset{g(x)}{x^2}$ $\quad \lim_{x\to\infty} \frac{10x}{x^2} = 0$

x^2 faster

or $10x$ slower

If x doubles $\quad \lim_{x\to\infty} \frac{x^2}{10x} = \infty$

$f(x)$ doubles

$10(2x) = 20x$ $\quad g(x), g(2x)$

$x^2 \quad (2x)^2 = 4x^2$

$g(2x) = 4 \cdot g(x)$

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$f(x) = x^2$

$g(x) = 5x^2 \quad g(2x) = 5(2x)^2 = 20x^2$

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Which function grows faster? (prove)

e^x, x^2 $\quad \lim_{x\to\infty} \frac{e^x}{x^2} = \lim_{x\to\infty} \frac{e^x}{2x} = \lim_{x\to\infty} \frac{e^x}{2} = \infty$

e^x faster

$\ln x, x, x^2$

$\lim_{x\to\infty} \frac{\ln x}{x} = \lim_{x\to\infty} \frac{\frac{1}{x}}{1} = \lim_{x\to\infty} \frac{1}{x} = 0$ $\quad \ln x$ slower

$\frac{\infty}{\infty}$

$\frac{x}{x^2} = \frac{1}{x} \to 0$ $\quad x$ slower than x^2

$\ln(2x) = \ln x + \ln 2$

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Show the functions grow at the same rate

$x, x+\sin x$

$$\lim_{x\to\infty} \frac{x+\sin x}{x} = \lim_{x\to\infty} \frac{1+\cos x}{1} \le \frac{1+1}{1}=2$$

$$\lim_{x\to\infty} 1+\frac{\sin x}{x} = 1+0=1$$

$\log_a x, \log_b x$

$$\lim_{x\to\infty} \frac{\log_a x}{\log_b x} = \lim_{x\to\infty} \frac{\frac{1}{x\cdot\ln a}}{\frac{1}{x\cdot\ln b}}$$

$$= \lim_{x\to\infty} \frac{x\ln b}{x\ln a} = \frac{\ln b}{\ln a}$$

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Transitivity of Growing Rates

Show the functions grow at the same rate by comparing both with x

$\sqrt{x^2+5}, (2\sqrt{x}-1)^2$

$$\lim_{x\to\infty} \frac{\sqrt{x^2+5}}{(2\sqrt{x}-1)^2} = \lim_{x\to\infty} \frac{2x\cdot\frac{1}{2}(x^2+5)^{-\frac{1}{2}}}{2\cdot\frac{1}{2}x^{-1/2}\cdot 2(2\sqrt{x}-1)^1}$$

$\frac{\infty}{\infty}$

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$$\lim_{x\to\infty} \frac{\sqrt{x^2+5}}{x} = \lim_{x\to\infty} \frac{\cancel{2x\cdot\frac{1}{2}}(x^2+5)^{-\frac{1}{2}}}{1}$$

$$= \lim_{x\to\infty} \frac{x}{\sqrt{x^2+5}}$$

same

$$\lim_{x\to\infty} \frac{\sqrt{x^2+\cancel{5}}}{x} = \lim_{x\to\infty} \frac{\sqrt{x^2}}{x} = \lim_{x\to\infty} \frac{x}{x} = 1$$

same

$$\lim_{x\to\infty} \frac{(2\sqrt{x}-\cancel{1})^2}{x} = \lim_{x\to\infty} \frac{4x}{x} = 4$$

Same

$$\frac{(2\sqrt{x}-1)^2}{(\sqrt{x})^2} = \left(\frac{2\sqrt{x}-1}{\sqrt{x}}\right)^2 = \left(\frac{2\sqrt{x}}{\sqrt{x}}-\frac{1}{\sqrt{x}}\right)^2$$

$$= \lim_{x\to\infty}\left(2-\frac{1}{\sqrt{x}}\right)^2 = 2^2$$

0

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Sequential vs Binary Searches

cut in half

best	1^{st}	7 100	best	1^{st}
worst	last	6 50		
		5 25		
ave	middle	4 13	$2^x=100$	
	$\frac{x}{2}$	3 7	$x=\log_2 100$	
linear		2 4		
		1 2	worst	
		1		

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